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Levels of Learning

Planning in Depth

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A FEW YEARS AGO there was considerable emphasis on fine gradations of arithmetic skills as the key to improving instruction. According to this point of view much of the lack of success children experienced in learning arithmetic was attributed to a too-rapid exposure to the "special difficulties" of a given operation. Curricula of this period provided teachers with elaborate gradations of "easy steps leading to the mastery of arithmetic."

Time and sad experience have brought about a re-evaluation of what such arithmetic gradations can and cannot do. As originally applied, this instructional principle represented a kind of "muscle theory" applied to the learning of arithmetic. As the rigor of the training was increased, children were supposed to develop "arithmetic muscles"—power to handle more difficult examples.

An arithmetic gradation, in the sense referred to, does not get at the heart of the learning problem because there is in it no provision for growth in the maturity of the thinking. In too many classrooms each "new step" is still initiated with the words spoken or implied, "Here is how we do examples like these." With this introduction the teacher launches into an "explanation" that employs the exact thought patterns and written procedures the youngster is expected to follow for the rest of his days. There is no provision for growth in intellectual maturity because there is no depth in the planning.

Pacing the Rate of Learning

We plan for different stages of maturity by providing for different levels of thinking in dealing with any given concept. Such planning provides an important basis for meeting individual differences within a given grade class because all the children will have access to some mode of attack on the work at hand. Such planning may make it possible to stretch out over several years the development of many of the more difficult concepts in arithmetic. This aspect of arithmetic planning has come to be known as *pacing* the rate of learning.

In the following paragraphs an illustration has been worked out to give substance to the methodological principle just reviewed. Most teachers will doubtless agree that ratio and proportion is high on the list of arithmetical topics difficult to teach. Yet the writer has seen primary grade children working understandingly with this concept at the manipulatory level. If it can be shown that primary grade pupils can use ratio and proportion as a way for dealing with cer-

tain social experiences, then we may reasonably conclude that other levels of intermediate maturity may be found that will be appropriate for more mature children preparing to work with the concept. The example to follow will seek to illustrate such a sequence.

A teaching plan has been roughly outlined to illustrate four levels of attack on the concept of ratio and proportion. The intellectual demands for operating at Level 1 are few. Many children in the primary grades will be able to deal with the concept at this level. It should be noted, however, that no grade level has been arbitrarily assigned because we are dealing with a stage of intellectual maturity and not physical maturity. We shall use the procedure appropriate to the capacity of the learner without regard to the chronological age. Levels 2, 3 and 4 will be introduced as the pupils show readiness for more mature modes of attack. By the ninth grade many pupils will be prepared to deal with ratio and proportion at Level 4. On the other hand it is to be expected that some pupils will never be able to operate successfully at this level. Such children will be encouraged to work at the highest level that they can handle successfully.

To save space all four procedures will be introduced with the same verbal problem. Only the maturity of the attack will be varied from section to section.

Illustrations of Maturity Levels

Verbal Problem

If 5¢ will buy two pieces of candy, how many pieces can be purchased for 20¢?

MATURITY LEVEL #1. The manipulatory level is excellent for children who are not able to do a great deal of reasoning at the verbal level.

Laboratory Devices: Circular discs to represent the coin series and small squares to represent the candy series. (It is not necessary to change materials with the subject-matter. Squares and discs can represent two different series no matter if the subject matter calls for miles traveled and gallons of gasoline used, or the number of chores completed in minutes of time.)

Procedure: This level of attack is based on the principle of one-to-one correspondence. First of all the children need to see that there are two series of things being used together—nickels and candy. The more nickels, the more pieces of candy. The problem starts the series (the ratio) and the children can set up the first arrangement as described in the problem.

$$0 = \text{nickel}$$

$$XX = \text{candy}$$

Similar groups are added until the number of nickels used equals the total sum of money to be spent.

0	0	0	0
XX	XX	XX	XX

The number of pieces purchased are counted. Twenty cents will purchase eight pieces of candy.

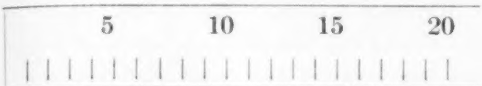
In a similar fashion the cost of 10 pieces of candy can be obtained. Children will then be asked to make new problems (verbalize), to ask the pertinent question, and work out the solution.

At the proper time a new ratio will be introduced and the process will be repeated.

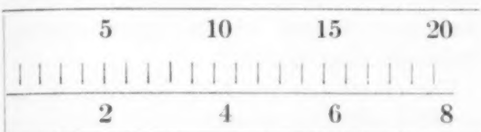
MATURITY LEVEL #2. The *transitory level*, as the name implies, represents an intermediate stage between the manipulatory level and the standard algorism (conventional form of writing the example). The work is partly symbolic and partly diagramatic. The structural element is emphasized—that is,

Laboratory Devices: Two paper strips (or blackboard drawings) are needed to represent the two different series. After the first few examples the children will be taught to draw the scales with paper and pencil.

Procedure: "This is a story about money and candy. The money increases in sets of fives and the candy by sets of twos. First we need to construct a five scale to show how the money increases." A counting scale for 5's is constructed



"Now a second scale must be prepared to show the pieces of candy. The scale must be drawn so that when it is laid under the five scale the 2 will fall directly under the 5. Why? (This is the ratio.) Now continue the scale in equal intervals."



The scale shows that twenty cents will purchase 8 pieces of candy.

Other examples may be worked and scales for the new ratios will be drawn. After a few examples have been worked in this fashion many of the pupils will be ready to shorten the process.

Modification of Procedure: A number of the children will quickly see that it is unnecessary to draw the scales to obtain the desired results. A writing of the corresponding points in the two series will do equally well.

$$\frac{5}{2} \qquad \frac{10}{4} \qquad \frac{15}{6} \qquad \frac{20}{8}$$

The results are fractions. The pupils may see that this discovery is consistent with past experience because a fraction is an expression of a comparison. Furthermore, some children will soon discover that the fractions are equivalent—each is reducible to the first.

New examples are worked applying the new discoveries. The bar in the fraction may be read as "for"—"5 for 2," "10 for 4," etc.

MATURITY LEVEL #3. The algorism represents a standardized approach to the solution of examples of a given type. The procedures are rather uniformly applied and the arrangement of the work follows a conventional pattern. The symbolic manipulations at this level seldom reveal much relationship to what would be done at the one-to-one correspondence level.

Laboratory Devices: None are used; the work is completely symbolic.

Procedure: The attention of the children is directed to the expression, "5¢ for 2." "What does it tell us?" It is a statement of *rate* or *comparison*—"five cents for two pieces of candy." Comparisons, we have learned, may be written as fractions:

$$\frac{\text{compare}}{\text{compare with}}$$

"In this example we would write 5. Why?"

"The more money that is spent, the more candy given in exchange. The numbers in the two series change together because for each additional nickel there will be two more pieces of candy. Any extension of the rate to new combinations of nickels and pieces of candy will always give a fraction reducible to 5.

$$\frac{5}{2}=\frac{10}{4} \qquad \frac{5}{2}=\frac{15}{6} \qquad \text{etc.}$$

"Problems in which a rate is being extended to new combinations are called *ratio* and *proportion*. What does the expression mean?" (If needed, an illustration may be drawn similar to the figure used in Level 1.)

"Verbal problems in ratio and proportion usually have a number missing that is needed to complete the proportion." Illustrate with problem about candy and nickels. In such cases a letter is used to show the missing term. "What question is asked in the following statement?"

$$\frac{5}{2}=\frac{20}{N}$$

"If both fractions have to be equal—the second reducible to the first—can you discover a way for finding the value of N?"

Children discuss the skills they have learned when reducing fractions to find a procedure applicable here.

$$\frac{5\times ?}{2\times ?}=\frac{20}{N} \qquad \frac{5\times 4}{2\times 4}=\frac{20}{8} \qquad \frac{5}{2}=\frac{20}{8}$$

As in the previous levels, new applications will be made by varying the social setting and the ratio.

MATURITY LEVEL #4. The more mature students will be capable of taking advantage of short cuts and equivalent operations to arrive at their solutions. This level will become the normal procedure for the typical ninth grader.

Laboratory Devices: None.

Procedure: The development will be very similar to that used in Level 3 down to the solution. After the statement with the missing term is written in the form

$$\frac{5}{2}=\frac{20}{N}$$

the pupil's thinking will be directed to the usefulness of changing fractions to common denominators to facilitate comparison. A common denominator for

$$\frac{5}{2} \text{ and } \frac{20}{N} \text{ is } 2N.$$

Then complete the change to obtain

$$\frac{5N}{2N}=\frac{40}{2N}$$

Now since the denominators are the same and the fractions are equal, we know that $5N$ must equal 40. Hence it follows that the value of N must be 8.

In practice the thinking may be reduced to "cross multiplication" to shorten the work and speed up the process.

$$\frac{5}{2}=\frac{20}{N}$$

$$5N=40$$

$$N=5$$

Summary

Without question, the development of the concept of teaching by levels of learning has become the most important development in arithmetic instruction in the past decade. Indeed its full implications in terms of curriculum development are yet to be explored. If the promise for the future is bright, the rewards for the classroom teacher here and now are immediate and generous—directly proportionate to his insight and willingness to experiment. Several of the more obvious dividends may be cited.

For classroom teachers instruction by levels of learning provides the best answer to the question, "How can the same course of study be covered by all pupils and all classes?" There is a line of communication available for reaching each intellectual level. Both slow pupils and bright pupils will progress from topic to topic when the time comes. The slow pupils will have a working knowledge of each topic and the ability to perform the most elementary computations but they are not expected to perform on levels they cannot possibly reach. The bright children, on the other hand, are challenged to find new algorisms, short cuts or equivalent operations that will save time in more complex number situations. This is the essence of differentiation of instruction in depth.

There are dividends, too, for the children. In the past many pupils have become discouraged from the lack of success. They lost hope either because the impossible was demanded of them or because they were systematically excluded from working on new and interesting topics that were "too difficult" for them. But the bright children frequently lost interest

too. The endless repetition of routine assignments "to keep them busy" until the slow pupils caught up usually killed their zest for inquiry and discovery. For both bright and slow children personal motivation and effort are dependent upon feelings of success and we cannot hope to tap these invaluable resources until we can provide adequate intellectual differentiation in instruction.

EDITOR'S NOTE. Perhaps the most significant change in our teaching of arithmetic in the past twenty-five years has come about through a better understanding of child development and what this means in terms of growth in learning. We no longer present to the child as his first experience with a topic the adult level of written algorism and expect him to memorize this so that he can use it in school and later as an adult. Rather, we view our work in school as a growth process from simple manipulative levels to the higher levels of working with abstract symbols. We believe that the understanding gained in this progression is in itself very valuable. We believe that each child should progress to the highest level at which he can function effectively. But we do want him to function efficiently and effectively. Society expects this of each of us and it is the job of the school to work toward that end. Dr. Moser has illustrated very well how we can work at various levels of maturity. It is our responsibility to do this with all phases of arithmetic. Sometimes the progression from one level to another is almost instantaneous and at other times it covers several years. With one pupil it may be easy and quick but with another it is difficult and time consuming.

A Mathematics Christmas Tree

For Christmas in our mathematics room we like to have a mathematical Christmas tree and, if possible, to vary the type from year to year. On our last tree the ornaments were mostly materials we use in mathematics. The tree was made of yardsticks, the base of wooden protractors, and the star of compasses. The ornaments on the bottom row were Hindu-Arabic numerals—zero through nine. The second row had circles, triangles, squares, and other plane figures. From the third row hung solids such as the pyramid, cone, cube, prism, and cylinder. Erasers, string, chalk, thumb tacks, and other useful materials were used for more ornaments. Lights were added to make the tree complete. The color scheme was red, green, and white. The shop teachers, an art teacher, and a student teacher cooperated with the pupils to make the tree possible.

Reported by ELIZABETH RAGLAND
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Whither in Arithmetic Teaching?

A Perceptive Perspective View

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ARE WE MAKING PROGRESS in the teaching of arithmetic? Yes, but . . . Looking back over the past thirty, or so, years, I become discouraged at the confusion and lack of direction. Two professional educators (invented by me) will give a somewhat exaggerated view of what I have in mind.

Have You Met a Dr. Skylark?

The first of these educators is Dr. Hermann Skylark, supervisor of elementary education. When I first knew Hermann, thirty years ago, he could not think of any bit of arithmetic except to break it into smaller bits. He searched for unit skills. He split skills before anyone ever split an atom. Long division was not long division. It was a cluster of sixty-seven unit skills. Hermann had his teachers nail down these skills—one by one. To him unit skills were the warp and woof of the magic carpet that carried the learner to quick perfection in arithmetic.

In time Dr. Skylark's Associative-Behavioristic psychology slipped. Hermann is not the sort to work with a losing cause. I talked to him at this time. His spirits were down, and he let his hair down in lamenting his sad fate. With hair down, he said, "To get anywhere in education, you must identify yourself with a catch-word and ride it for all it is worth." Hermann had staked his all on unit skills. He has ridden hard. That nag had let him down.

The Tenth Yearbook of the National Council of Teachers of Mathematics came out about this time. This proved a lifesaver for Hermann. In it he found two nags that he could ride simultaneously: Activity Program, and the New Psychology. He fused them. He sent out directives that teachers use arithmetic time to plant bushes on vacant lots—that they quit trying to teach arithmetic and begin to teach "life, discovery, and nature through arithmetic." He said that he had a firm grip on the reins and the road ahead was clear. He was well on his way, he said, to the pot of gold at the end of arithmetic rainbow. He said the past is all darkness and error—that if he only had the time he would write a series of Activity-New Psychology texts that would electrify arithmetic for generations to come. It is needless to say that Dr. Skylark never mentioned unit skills now. In a coy way, he passed out the word that he had been a *gestalt* pioneer.

I saw Hermann ten years later. His hybrid nag, Activity-New Psychology Arithmetic, was not so sturdy as once she had been. He was seeking a new mount: a mount that would put him again in the van of pedagogic progress. Borrowed nags had let him down twice. This time he would make one that was really his own. He would put things together by formula—to appeal to common sense and at the same time catch those who believe in magic. This was his formula:

60% of the new nag would be methods, principles, superstitions, and traditions that the average teacher already accepts.

20% would be an idea—any idea that is riding high at the time: activity arithmetic, creative arithmetic, discovery arithmetic, experimental arithmetic, any kind of arithmetic . . . for

the slow pupil, for the average pupil, for the superior pupil, for any kind of pupil. On only one point was he sure. Unit skills had to go. Instead of breaking bits into smaller bits, he would weld bits into patterns. His slogan would be: "Kids don't get it in bits; give it to them in gobs." Make them work for it.

Another 10%, by formula, would be given to disowning any theories that are under a cloud. An educational leader cannot afford to drag dead nags around with him.

The final 10%, by formula, would be away out front in something. This had to be a little obscure—to confuse and impress the pedagogic public. Dr. Skylark worried a lot on this item—even dreamed about it. In his dream he saw numbers spelled in order. In dreamland, no mention was made of odd and even numbers. The professional discussion was on consonant and vowel numbers. Consonant numbers were taught first. Any process involving a vowel number, a one or an eight to be specific, had to be deferred till the child was more mature. Hermann was deeply impressed by the dream. His first act on awakening was to start action research to confirm the new theory. He confided in me, "This deal is really new. It will put me two jumps ahead of anyone else in the race."

Hermann's new nag was ready now, except for a name. Mr. Skylark suggested "Miners' Math" for the child had to dig it out. Hermann vetoed this. Son Elmer suggested "Bootstrap Arithmetic," for that is the way the kid lifted himself. Elmer was sent to bed.

Hermann decided to call his new nag "Skylark's Rocket." What could be more appropriate! A skylark—the bird—rises vertically without even the help of bootstraps. Dr. Skylark said that with his new program pupils shoot upward in arithmetic with confidence and enthusiasm.

The last time I saw Hermann he was organizing workshops, so that all his teachers might get astride Skylark's Rocket and shoot upward with confidence and enthusiasm.

How Many Dr. Otts Have We?

The second of my invented educators is a comparatively young man. Only a few years ago, he was in the elementary school himself. Everyone called him Tommy. Tommy had his best foot foremost—in everything except school work. . . . This same Tommy is now Dr. Thomas R. Ott, Professor of Elementary Education—with his best foot foremost.

Dr. Ott began his professorial duties with a lucky find—the preface to Warren Colburn's Intellectual Arithmetic, published in 1821. This preface started brain cells working toward a professional text, *Teachers Arithmetic.* Dr. Ott wove two principles into his book.

First, a pupil learns only what he discovers for himself. In Dr. Ott's experimental school, pupils discover not one way but seven ways to divide a whole number by a fraction; discover not one way but nine ways to multiply with a 2-figure multiplier; discover not one way but thirteen ways to check addition.

Teachers Arithmetic, Dr. Ott's book, told and showed that the "tell and show" method is outmoded. College students wondered: "Why does he tell and show us not to tell and show?" "Does he think the pupils we teach are smarter than we are?"

Teachers Arithmetic, Dr. Ott's book, states categorically that you learn only what you discover. College students wondered: "If the discover method is so powerful, why cannot college students discover the discover method without all this tell and show?"

The second principle in Dr. Otto's book is that teachers emphasize right answers too much. Dr. Ott never got an answer exactly right, and this had not handicapped him at all. The book says that to

insist on right answers is to pour sand into the gears of creative learning. Dr. Ott will give his genius to something bigger than sums and products—something bigger than remainders and quotients. He will make arithmetic education into the modern science. He prides himself on his precise definitions and sharp distinctions. A teacher in his experimental school asked Johnny, "How many 8's are in 34?" Dr. Ott took the teacher aside quickly. He said, "Ridiculous question. They were asking that when I was in school. We are building a modern science. The modern question is, How many 8's are equal to 34? The modern answer is, Four 8's are equal to 34."

Johnny goes home now with 4 8's equal to 34 on Monday, 4 8's equal to 37 on Tuesday, 4 8's equal to 35 on Wednesday, 4 8's equal to 39 on Thursday, and 4 8's equal to 33 on Friday. Johnny's mother wonders. She went to school when 4 8's were 32 every day of the week.

Johnny's father teaches a science in the college—is a colleague of Dr. Ott. The father is amused with Tommy Ott. The musing goes like this: Tommy R. Ott! Tommy ROtt! Tommyrot! How silly can you get! How long will it be till you get that better job in the big university, from which you can beam your precise definitions and sharp distinctions on more and more teachers! Strains out the gnat. Swallows the whale. . . . But Tommy Ott is not the sort to get into a rut. Next year he will strain out bigger gnats and swallow bigger whales.

More than sixty years ago, John Dewey expressed concern about the "aimless swing of the scholastic pendulum . . . followed by equally unreasonable swing to other extreme." He said this about arithmetic. The arithmetic pendulum still swings. Today's oracle discredits what was said yesterday. Someday arithmetic education will stand where natural science stood when Newton lived. A necessary step is to close the chasm between yesterday and today.

Newton built upon the work of his predecessors: "If I have seen farther than Descartes, it is because I stood on the shoulders of giants." There have been giants in education. There are capable workers—possibly giants—in education now. When the pendulum swings in education, we discard the good with the bad—throw out the baby with the bath. Ideas, principles, theories that are basic to any solid advance in arithmetic education lie buried in the graveyards of dead isms. We must seek and save every good thing even though it may be associated with a program that we cannot accept as a whole. When arithmetic education is tied securely to the work of the giants, it will not be tossed about in petty brainstorms of Skylarks and Tommyrots.

An arithmetic authority de-emphasizes drill till the classroom teacher fears even to pronounce the word. A few years later the same authority emphasizes the need for drill. The teacher is confused. She lacks the perspective to carry on a balanced program amidst the aimless swings.

You may say, the teacher must learn not to accept a theory as magic just because a big name—or little name—says that it is; not to reject a plan just because it is old—or new. I say that a grave responsibility rests on those in leadership positions—not to mislead, not to confuse the teacher who trusts them.

EDITOR'S NOTE. Dr. Osborn has done a neat job in accentuating the swing of the pendulum in arithmetic teaching and in caricaturizing the types of people who often exert an undue influence upon the public schools. Fortunately our larger teaching population is slow moving and does not accept completely the leadership of the zealots. It is this same factor of slow motion however that keeps general teaching practice about twenty years behind the better present practices. How best can we move forward safely and sanely and yet be sure that we are progressing? What should be done in terms of the content of arithmetic and its placement in curriculum? What new methods of teaching and modes of learning are so sound that they should be shouted from the housetops? THE ARITHMETIC TEACHER exists so that ideas and views can be exchanged.

Non-Pencil-and-Paper Solution of Problems

An Experimental Study

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FREQUENTLY ONE HEARS the argument that an excessive amount of activity with pencil and paper occurs when children attempt to solve arithmetical verbal problems. One further hears the contention that extreme emphasis on the pencil-and-paper type of arithmetic has created the following undesirable practices and situations in the arithmetic program: complete lack of, or too little use of, mental arithmetic; careless habits, involving division of attention, resulting from continuous presentation of the printed statement of the problem while solving it with pencil and paper; failure to use methods of handling arithmetical situations that approximate more nearly methods of handling similar quantitative situations encountered in other areas of learning and in the everyday affairs of life; evaluation of a child's ability in arithmetic almost entirely upon the basis of pencil-and-paper work; limited use of different sensory modes of presentation of work in arithmetic; and complete lack of, or less efficient use of, "short-cuts."

Since there is much discussion but little evidence concerning the effects of the neglect of non-pencil-and-paper arithmetic, the following study seemed appropriate.

Purpose and General Plan of the Study

The experiment was designed to study the effectiveness of a non-pencil-and-paper method of solving verbal problems in arithmetic. The effectiveness of this non-pencil-and-paper method was determined by comparing the performance of a group of children who had practiced solving, without the use of pencil and paper, verbal problems not presented continuously during an allowed solution time with the performance of a group of children who had practiced solving, with the use of pencil and paper, the same verbal problems presented continuously during the solution time allowed for the non-pencil-and-paper group.

The children in the non-pencil-and-paper group were given, during their practice periods, twenty-five seconds to read (study or look at) the printed statement of the problem. At the end of the reading time the printed statement of the problem was removed from the pupil's sight and he was given thirty-five seconds to finish thinking out the solution of the problem and to record with pencil and paper only the answer. Thus, a specified total time of one minute was allowed for consideration of each problem. This non-pencil-and-paper method was a type of "mental" arithmetic.

The pupils who used the pencil-and-paper method of solving problems during their practice periods were allowed the same time limits for consideration of each problem as were allowed the non-pencil-and-paper group; however, the children in the pencil-and-paper group were permitted to see the printed statement of the problem during the entire time (one minute) allowed for consideration of a problem. The pupils in the pencil-and-paper group were instructed to use pencil and paper in solving the problems. In fact, they were asked to show the main calculations performed in arriving at the answer to each problem solved in the practice periods.

The word-problems used by both groups of children were presented in small booklets designed specifically for the experiment. The problems for the practice periods, as well as those on each test, were the same for both groups. The problems used in the practice periods were always printed on an odd-numbered page of the booklet. The even-numbered pages in the booklets for the non-pencil-and-paper group were left blank, except for space indicated for the answer to the problem read on the preceding page and for directions reminding the child to do no figuring with pencil and paper. Each problem in the practice booklets used by the pencil-and-paper group was printed on two separate pages. On the odd-numbered page where the child encountered the problem for the first time, he was directed to read or study the problem but to do no figuring. At the end of the twenty-five seconds allowed for reading or studying the problem, the pupil was instructed to turn to the next page (an even-numbered page) to solve the problem with pencil and paper, showing the main calculations. The printed statement of the problem occurred again on the even-numbered page.

The subjects in this experiment were 257 sixth-grade pupils in 10 different classrooms of four school systems in Iowa. The pupils in each classroom were randomly divided into two groups that were approximately equal in size. This random selection of the pupils in the various classrooms was accomplished by passing out, at the beginning of the first practice period, booklets that contained problems for practice by the non-pencil-and-paper group that were alternately arranged with booklets containing problems for practice by the pencil-and-paper group. The type of booklet received at this first practice period determined the group to which a child was assigned for the entire experiment. There were 133 children in the pencil-and-paper group and 124 in the non-pencil-and-paper group. All pretesting was done before the two practice groups were established.

The administration of the practice work to the two different groups within a classroom posed the chief difficulty of administering the work of the experiment but this situation was met in the following manner. After the general directions were cared for, the teacher said to both groups, "Turn to page one and read or study the problem." At the end of twenty-five seconds, the teacher said, "Turn to page two and solve the problem." When the non-pencil-and-paper group turned to page two, they did not find the problem, which was on page one, restated. These pupils of the non-pencil-and-paper group were, also, required to solve the problem "mentally" and to record only the answer on page two. When the children in the pencil-and-paper group turned to page two, they found the problem, which was on page one, duplicated on page two. The children in the pencil-and-paper group solved the problem with pencil and paper on page two, showing their main calculations. At the end of the thirty-five seconds allowed for the solution of the problem, the teacher told both groups to turn to page three and read the next problem. Thus, the same procedure used in the first problem was begun again. This procedure was followed for each problem in the different practice booklets.

The two chief differences in the procedures for the two groups were: (1) non-use of pencil and paper versus use of pencil and paper; and, (2) a brief presentation of the printed statement of the problem followed by a solution time during which time the pupil did not have access to the printed statement of the problem versus a continuous presentation of the printed statement of the problem during the entire time allowed for reading and solving the problem.

Verbal problems in arithmetic constituted the material used in this study. The problems constructed for this experi-

ment were similar in nature and difficulty to the problems found in sixth-grade arithmetic textbooks. Since a uniform time was allowed for reading or studying each problem in the practice exercises and in some of the tests, all the problems contained approximately the same number of words. It was fully realized that two verbal problems of the same length in reference to the number of words they contained were not necessarily comprehended equally well during the same reading time limits; however, the above procedure was thought best for an experimental study of this nature. Most of the problems used in the practice material and in the tests were subjected to tryouts in fifth and sixth-grade classes.

The directions for administering the practice materials and all the tests were prepared for the cooperating teachers. Materials were delivered to the cooperating teachers in person by the experimenter at which time conferences between the experimenter and the teacher were held. The cooperating teachers were encouraged not to inject any additional directions or motivation. The experiment was conducted at the same time of the school year for all the children.

The practice materials and tests were returned to the experimenter who did all the scoring in order to eliminate variations which might result if different persons did the scoring.

Comparison of the Performance of the Two Groups

The effectiveness of the two methods of practice in solving verbal problems was measured by the use of four criterion tests: non-pencil-and paper test; pencil-and-paper test; orally stated test solved by any method; and, a type of speed test solved by any method. For each of these criterion measures, a pretest and a final test were given the two groups. In each case the difference between the mean initial score and the mean final score for one group was compared with the difference between the mean initial score and the mean final score for the other group. Equivalent pretests and final tests were not thought necessary in this study.

To test the hypothesis that the over-all mean gain or change of performance for the pencil-and-paper group was equal to the over-all gain or change in performance of non-pencil-and-paper group, the *t*-test for related measures was used to test the null hypothesis.

Tables I, II, III, and IV show the results of the calculations of the *t*'s for the four criterion tests.

As can be seen from the foregoing Tables, the *t*-test of the difference of the over-all mean gain or change of performance of the two groups from the pretest to the final test for each of the criterion measures was not statistically significant at the five per cent level of confidence.

The data obtained suggest that differences in mean gain or changes in performance on the criterion tests favored the group who had used during the practice periods a method which was the same as, or similar to, the method required in the respective criterion tests. For example, five days of practice in putting down the main calculations, perhaps, gave the pencil-and-paper group an advantage over the non-pencil-and-paper group when both groups were measured by tests requiring the main calculations to be shown. Likewise, when the two groups were measured by tests requiring non-pencil-and-paper solutions of problems not continuously presented during the allowed solution time, the non-pencil-and-paper group, no doubt, had an advantage over the pencil-and-paper group.

The noted trend indicates that the maximum improvement in ability to solve verbal problems by either of the two methods used in this study could, perhaps, best be accomplished by practice on the respective method. This trend, however, was not statistically significant.

TABLE I

T-TEST OF THE SIGNIFICANCE OF THE DIFFERENCE IN THE MEAN GAIN OR CHANGE IN THE PERFORMANCE OF THE TWO GROUPS ON NON-PENCIL-AND-PAPER TEST

School	Mean Score on Pretest		Mean Score on Final Test		Mean Gain from Pretest to Final Test		Difference in Mean Gain
	C*	E†	C	E	C	E	(E—C)
A	5.38	5.69	6.31	6.85	.93	1.16	.23
B	4.80	5.69	2.80	4.00	−2.00	−1.69	.31
C	7.63	6.76	6.79	6.41	− .84	− .35	.49
D	7.00	7.60	6.73	8.00	− .27	.40	.67
E	4.85	7.36	4.46	6.18	− .39	−1.18	− .79
F	6.35	7.38	4.82	6.15	−1.53	−1.23	.30
G	4.56	5.27	3.63	6.00	− .93	.73	1.66
H	8.42	7.60	6.08	5.10	−2.34	−2.50	− .16
I	9.00	8.55	8.50	8.36	− .50	− .19	.31
J	6.58	6.91	5.50	5.64	−1.08	−1.27	− .19

$\overline{D} = .28$

Est $\sigma_{\overline{D}} = .202$

$$t = \frac{\overline{D}}{\text{Est } \sigma_{\overline{D}}}$$

$$t = \frac{.28}{.202}$$

$t = 1.401$‡

$\overline{D}$ indicates the mean of the differences in mean gain.
$\sigma_{\overline{D}}$ indicates the standard error of the mean.
t indicates the ratio of a normally distributed variate to its estimated standard error.
* Control Group (Pencil-and-paper Group).
† Experimental Group (Non-pencil-and-paper Group).
‡ Not significant at the five per cent level of confidence.

TABLE II

T-TEST OF THE SIGNIFICANCE OF THE DIFFERENCE IN THE MEAN GAIN OR CHANGE IN THE PERFORMANCE OF THE TWO GROUPS ON PENCIL-AND-PAPER TEST

School	Mean Score on Pretest		Mean Score on Final Test		Mean Gain from Pretest to Final Test		Difference in Mean Gain
	C*	E†	C	E	C	E	(E—C)
A	6.77	6.54	5.92	5.46	− .85	−1.08	− .23
B	4.30	4.77	3.20	3.92	−1.10	− .85	.25
C	7.26	6.35	6.32	5.35	− .94	−1.00	− .06
D	6.00	6.60	7.55	7.30	1.55	.70	− .85
E	5.08	7.64	4.08	6.27	−1.00	−1.37	− .37
F	6.41	7.15	6.59	7.31	.18	.16	− .02
G	4.00	5.53	4.69	5.00	.69	− .53	−1.22
H	9.42	8.00	7.67	7.90	−1.75	− .10	1.65
I	9.70	9.45	8.20	7.45	− .50	−2.00	−1.50
J	6.33	7.00	6.25	6.36	.08	− .64	− .72

$\overline{D} = -.307$

Est $\sigma_{\overline{D}} = .279$

$$t = \frac{\overline{D}}{\text{Est } \sigma_{\overline{D}}}$$

$$t = \frac{-.307}{.279}$$

$t = 1.100$‡

* Control Group (Pencil-and-paper Group).
† Experimental Group (Non-pencil-and-paper Group).
‡ Not significant at the five per cent level of confidence.

TABLE III

T-TEST OF THE SIGNIFICANCE OF THE DIFFERENCE IN THE MEAN GAIN OR CHANGE IN THE PERFORMANCE OF THE TWO GROUPS ON ORALLY STATED TEST

School	Mean Score on Pretest		Mean Score on Final Test		Mean Gain from Pretest to Final Test		Difference in Mean Gain
	C*	E†	C	E	C	E	$(E-C)$
A	7.23	7.46	7.46	7.85	.23	.39	.16
B	5.10	5.69	4.20	6.00	− .90	.31	1.21
C	8.58	7.24	8.16	6.94	.42	− .30	.12
D	7.55	7.50	8.55	8.00	1.00	.50	− .50
E	5.69	9.09	5.92	7.82	.23	−1.27	−1.50
F	8.64	8.38	7.12	7.46	−1.52	− .92	.60
G	5.69	7.07	6.75	7.27	1.06	.20	− .86
H	9.92	8.80	9.17	8.50	− .75	− .30	.45
I	10.60	9.00	7.90	7.09	−2.70	−1.91	.79
J	9.00	7.64	8.42	8.36	− .58	.72	1.30

$\bar{D}=.177$

$\text{Est } \sigma_{\bar{D}}=.252$

$$t=\frac{\bar{D}}{\text{Est } \sigma_{\bar{D}}}$$

$$t=\frac{.177}{.252}$$

$$t=.702‡$$

* Control Group (Pencil-and-paper Group).
† Experimental Group (Non-pencil-and-paper Group).
‡ Not significant at the five per cent level of confidence.

TABLE IV

T-TEST OF THE SIGNIFICANCE OF THE DIFFERENCE IN THE MEAN GAIN OR CHANGE IN THE PERFORMANCE OF THE TWO GROUPS ON SPEED TEST

School	Mean Score on Pretest		Mean Score on Final Test		Mean Gain from Pretest to Final Test		Difference in Mean Gain
	C*	E†	C	E	C	E	$(E-C)$
A	7.85	7.54	10.54	9.23	2.69	1.69	−1.00
B	5.70	7.38	5.20	7.15	− .50	− .23	.27
C	9.86	9.35	10.79	9.59	.93	.24	− .69
D	9.73	11.10	10.64	9.60	.91	−1.50	−2.41
E	6.15	12.45	7.77	11.73	1.62	− .72	−2.34
F	10.76	10.23	10.41	10.23	− .35	.00	.35
G	7.94	9.47	8.06	9.07	.12	− .40	− .52
H	10.50	9.10	13.33	10.80	2.83	1.70	−1.13
I	11.90	11.18	13.80	12.73	1.90	1.55	− .35
J	11.42	10.00	10.08	9.91	−1.34	− .90	1.25

$\bar{D}=-.657$

$\text{Est } \sigma_{\bar{D}}=\ .363$

$$t=\frac{\bar{D}}{\text{Est } \sigma_{\bar{D}}}$$

$$t=\frac{-.657}{.363}$$

$$t=1.899‡$$

* Control Group (Pencil-and-paper Group).
† Experimental Group (Non-pencil-and-paper Group).
‡ Not significant at the five per cent level of confidence.

A comparison of the difference of the mean gain or change in performance of the pupils in the two groups at different levels of ability showed the same general trends as those shown when all the children of the whole group were considered.

Analyses of the pupils' performances on the five practice exercises were made. The comparison of the over-all mean scores of the two groups on the different practice exercises is presented in Table V. The *t*-test of the difference in the over-all means for the two groups on each of the practice exercises was applied. From the data given in Table V, it can be seen that the advantage favored the pencil-and-paper group for each of the five practice exercises; however, only the difference in Practice Period I was statistically significant at the five per cent level of confidence. Although the pencil-and-paper group consistently made higher scores on the practice exercises, the pencil-and-paper method was not significantly more effective in solving the problems used in this study than was the non-pencil-and-paper method after the pupils had some practice with this latter method. There was some indication that the difficulty of the problems was a factor which determined the magnitude of the difference in the means of the two groups on the practice exercises.

TABLE V

COMPARISON OF THE OVER-ALL MEAN SCORES OF THE TWO GROUPS ON THE DIFFERENT PRACTICE EXERCISES

Group	Practice Period				
	I	II	III	IV	V
Control*	7.51	8.57	8.00	8.26	6.66
Experimental†	6.48	8.52	7.27	7.43	5.91
Difference	1.03	.05	.73	.83	.75

* Pencil-and-paper group.
† Non-pencil-and-paper group.

Since there will be a need, on numerous occasions, to handle without the use of pencil and paper quantitative situations met in the school and outside the school, the statistical analysis of the performance of the two groups on the practice exercises certainly emphasized the need for giving children training with a non-pencil-and-paper method of solving verbal problems not continuously presented during the allowed solution time.

Experiment II

A second experiment that used, in general, the same two methods of solving the same verbal problems as found in the first experiment was conducted. In Experiment II, however, all the pupils in each classroom were concerned with only one method. Ten different classes of children in six school systems in Iowa were paired on the basis of their general average in arithmetic tests of the *Iowa Every-Pupil Tests of Basic Skills*. The method used by each of the paired schools was determined by the flip of a coin. There were 163 pupils in the pencil-and-paper group and 145 in the non-pencil-and-paper group.

One other important modification was made for Experiment II. The problems in the practice exercises were not timed individually for the pencil-and-paper group as was the case in Experiment I; but, the total time allowed the pencil-and-paper group for each practice exercise was equal to the total time necessary for the non-pencil-and-paper group to consider individually each problem in the practice exercises as was done by this group in Experiment I.

Since the behavior of the two groups in Experiment II paralleled, in general, the results of Experiment I, the satistical analyses and the results of the *t*-tests are not herein shown.

General Conclusions

Subject to the limitations in this study and the usual hazards of making generalizations, the following tentative conclusions were drawn from the data:

1. The evidence did not indicate the superiority of either method of solving the

verbal problems used in this study. There was a tendency for a given group to solve problems better than the other group when the method of solution required in a criterion test was more similar to the method the given group used in the practice exercises.

2. Pupils trained in the non-pencil-and-paper method tended to use pencil and paper less on exercises and tests where the use of pencil and paper was optional than did the pupils trained in the pencil-and-paper method.

3. Neither method appeared to be more effective in producing gain or change in ability to solve by any method orally stated problems.

4. As was shown in practice exercises, pupils solving verbal problems with the use of pencil and paper solved correctly more of the problems under the conditions specified in this study than did the pupils solving the same problems without the use of pencil and paper; however, the advantage became much less as the non-pencil-and-paper group became accustomed to the non-pencil-and-paper method.

5. As the number of steps required in the solution of problems increased, the use of pencil and paper appeared to be more essential for successful performance.

6. Neither method was more effective with a given level of general ability.

7. Teachers and pupils enjoyed the non-pencil-and-paper method of solving verbal problems; however, they believed both methods should be employed in the regular instruction in arithmetic.

8. The cooperating teachers believed changing the methods of solving verbal problems would motivate the children's work.

Suggestions

In the light of the results of this study, the following suggestions for better materials and instruction in problem solving should receive serious consideration.

1. In each manual for an arithmetic textbook there should be distributed throughout the manual pages of problems which the teacher could use for orally stated tests or practice exercises.

2. Teachers and/or schools should construct sets of problems in booklet forms such as the ones in this study, which could be used year after year for practice with a non-pencil-and-paper method of solving verbal problems.

3. Teachers and/or schools should print problems individually on one side of cards which could be used by the children in practice with a non-pencil-and-paper method of solving verbal problems or with practice exercises in which the problems are timed individually.

4. Tachistoscopic slides should be constructed and used for timed exposures of individual problems or sets of problems, if such equipment is available.

5. Teachers should set aside certain pages of verbal problems in the textbook which would be solved "mentally."

EDITOR'S NOTE. Dr. Petty calls to our attention the need for oral-mental arithmetic because so much of the mathematics we encounter outside of school requires a judgment response which should frequently be formed without pencil and paper. Many of these situations are not presented by the written word; they may be oral or directly visual. In his experiment it was found that pupils tended to increase in problem solving ability of the type they practiced. This is in harmony with previous experiments. Since the list of problems used in the practice periods and in the tests are not available it is not possible to judge whether these problems tended to be of a type more suited to written procedures or to oral-mental procedures. The problems we encounter seem to fall into two categories: (A) those that we should be able to do mentally and (B) those that most people should probably do with pencil and paper. Of course, these categories are very flexible. An oral problem for John may well be a written problem for Sam and a written problem for Sam today should be an oral-mental one for him perhaps next week or next month. Because children differ so much in their reaction rates (both mental and physical) it is difficult to set an optimum time limit for a particular problem. Although the Petty experiment did not yield significant results his basic thesis seems valid. We should give the types of practice and experience in school that will be most useful both in and out of school. Very often the modes of solution used mentally are different from those used with pencil and paper. We need experience with both.

Why "Indent" in Multiplication?

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We may be doing more harm than good when we teach children deductively to use many of the traditional techniques of computation. "Carrying" in addition; "borrowing" in subtraction; "indenting" in multiplication; "multiplying-subtracting-bringing down" in division; "canceling" in multiplication of fractions; "inverting and multiplying" in division of fractions; "counting decimal places" in multiplication of decimals; and "moving decimal points" in division of decimals are cases in point. To illustrate why this may be so, let us consider "indenting" in multiplication, although any one of the other techniques mentioned could serve equally well as an example of deductive teaching.

The traditional method of multiplying with a 2-digit multiplier goes something like this:

$$\begin{array}{r} 68 \\ \underline{\times 32} \\ 136 \\ \underline{204} \\ 2176 \end{array}$$

Think: Two 8's are 16, write 6, carry 1; two 6's are 12 and 1 are 13. Three 8's are 24, put the 4 under the 3, carry 2; three 6's are 18 and 2 are 20. Bring down the 6; 3 and 4 are 7; 1 and 0 are 1; bring down the 2.

Without going into a discussion of the "carrying" technique at this point, let us consider the practice of "putting the 4 under the 3," or "indenting."

The writer has heard teachers explain to children the reasoning behind this practice of "indenting" somewhat as follows:

"You multiply 8 ones by 3 tens, making 24 tens, which is 4 tens and 2 hundreds, so the 4 goes in the 10 column under the 3. It is 'indented' one place."

Children usually listen politely to such an explanation, impatient to go ahead with the actual computing. Very few children, however, in the writer's experience really understand the ones-tens-hundreds type of explanation for this and many other kinds of computational processes.

Such types of explanation seem arbitrary and tend to contribute to the mystery which many children (and youth and adults, for that matter) find in arithmetic.

If one were to add the two numbers, 136 and 2040, one would ordinarily write the numbers like this, $\begin{array}{r}136\\ \underline{2040}\end{array}$ or like this, $\begin{array}{r}2040\\ \underline{136}\end{array}$ and then proceed to add. It is inconceivable that the numbers would be written like this, $\begin{array}{r}136\\ \underline{204}\end{array}$. Yet, this latter way is precisely how we tell children to write the numbers that we want them to add when they multiply such numbers as 32 and 68.

Ask the child who is performing this multiplication to *read* the two numbers that he is adding. What would he read? "One hundred thirty-six and two hundred four?" But that would not be correct. Yet how should he be expected to read, "Two thousand forty," for a number that appears like this, 204. If you tell the child that 204 is really 2040, he has a perfect right to ask, "Then why don't I write it as it really is?"

And indeed, why shouldn't he write it as 2040?

Why should we introduce such a technique as "indenting" to children before

they are ready to understand it? Why should we involve children unnecessarily in the intricacies of our number system? Why should we run the risk of making arithmetic an exercise in the manipulation of mysterious and meaningless symbolisms? (Perhaps one of the major reasons why engineering and the sciences have difficulty in recruiting college students into their extremely understaffed ranks is the fear and dislike of mathematics engendered in so many children and youth.)

"Indenting," as such, is a perfectly acceptable technique of computation for the person who knows what he is doing. It is acceptable if he knows that he is *not* writing the zero at the end of the second partial product instead of *writing* it. It is acceptable as a time-saver, as a short cut, as a trick of computation.

Are elementary school children capable of learning the process of "indenting" as a time-saver, as a short cut, as a trick of computation? Yes, some are capable of it, but others would take *more* time in trying to learn to *save* time than the effort is worth. For them "indenting" would be a waste of time.

Undoubtedly, the children who *discover* the trick of "indenting" for themselves would be the most likely ones to understand *why* "indenting" is possible. It would seem, therefore, that the best kind of teaching would be inductive teaching, the kind which helps children to discover short cuts, tricks, and reasons themselves. The poorest kind of teaching might very well be the deductive type, the kind that *tells* children the techniques they must use and *tells* them why the technique works.

Children Can Discover

Teachers can best lead children to the discovery of "indenting," and other short cut tricks as well, by introducing them to increasingly complex multiplication processes gradually, step by step, in planned sequence. One type of process should follow from the preceding one and lead to the next. The child should understand one type of computational process and acquire facility in computing that type before undertaking the next.

A sequence of computational steps that can help children discover tricks in multiplication and reasons why the tricks work is presented below. (How children can be led to the discovery of techniques and tricks in addition, subtraction, and division of whole numbers and in the manipulation of fractions and decimals is discussed by the author elsewhere.[1])

It should be borne in mind that some children require more time than others to understand and gain facility in the various types of computation in a sequence. They might spend considerable time on one type and little time on another. Some children grasp new steps without the help of the teacher, applying, unaided, principles already acquired. Others have to approach each step with careful guidance from the teacher. The teacher should know the different capabilities of his children and guide them accordingly.

It should also be borne in mind that a multiplication algorism has no meaning for the child unless he can visualize what the symbolism represents—the combining of equal groups of objects. For $\begin{array}{r}4\\ \times 3\\ \hline\end{array}$, for example, he visualizes 3 groups of 4 objects; for $\begin{array}{r}28\\ \times 5\\ \hline\end{array}$, he visualizes 5 groups of about 28 objects.

The sequence of steps in multiplication follows:

1. After developing the meaning of multiplication (the combining of equal groups of objects) and after learning to represent the combining of equal groups by the appropriate symbols

[1] J. Allen Hickerson. *"Guiding Children's Arithmetic Experiences,"* Prentice-Hall, Inc., New York. 1952.

$\left(\text{e.g., } 2\times 8 = \begin{array}{r}8\\ \times\underline{2}\end{array}\right)$ *the child should memorize the 100 basic combinations* from $\begin{array}{r}1\\ \times\underline{1}\end{array}$ to $\begin{array}{r}10\\ \times\underline{10}\end{array}$.

2. *Multiply multiples of 10 by 1-digit numbers.*

$$\begin{array}{r}30\\ \times 2\\ \hline 60\end{array} \text{ can be developed from } \begin{array}{r}30\\ +30\\ \hline 60\end{array} \text{ and}$$

$$\begin{array}{r}40\\ \times 3\\ \hline 120\end{array} \text{ can be developed from } \begin{array}{r}40\\ 40\\ +40\\ \hline 120\end{array} \text{ etc.}$$

$$\begin{array}{r}30\\ \times 2\\ \hline 60\end{array} \text{ can also be developed from } \begin{array}{r}3\\ \times 2\\ \hline 6\end{array} \text{ and}$$

$$\begin{array}{r}40\\ \times 3\\ \hline 120\end{array} \text{ can also be developed from } \begin{array}{r}4\\ \times 3\\ \hline 12\end{array}$$

When practicing giving answers to this type of multiplication the child should aim to respond orally with about the same facility as he responds to the basic 100 combinations. Moreover, at this stage he computes from left to right not right to left. For $\begin{array}{r}70\\ \times\underline{8}\end{array}$, for example, he responds "560," not "eight 0's are 0, eight 7's are 56." He should be led to discover the trick of saying, "Eight 7's are 56" and then affixing the 0.

3. *Multiplying a 2-digit number by a 1-digit number; the product of the ones not exceeding 9.*

From previous experiences with numbers the child should know that the number 21 is composed of 20 and 1, 43 is composed of 40 and 3, etc. Likewise, he should know that when he adds such numbers as $\begin{array}{r}21\\ +\underline{21}\end{array}$ he can think: 20 and 20 are 40, 1 and 1 are 2; or he can add from right to left, 1 and 1 are 2, 20 and 20 are 40.

$$\begin{array}{r}21\\ \times 2\\ \hline 40\\ 2\\ \hline 42\end{array} \text{ can, therefore, be developed from } \begin{array}{r}21\\ +21\\ \hline 40\\ 2\\ \hline 42\end{array}$$

$$\begin{array}{r}21\\ \times 2\\ \hline 2\\ 40\\ \hline 42\end{array} \text{ and can be developed from } \begin{array}{r}21\\ +21\\ \hline 2\\ 40\\ \hline 42\end{array}$$

At this point the child can be helped to discover another trick or short cut—if he does not think of it himself.

When adding 21 and 21 from right to left, the sub-totals 2 and 40 are to be added. This he can learn to do mentally. His written computation then looks like this $\begin{array}{r}21\\ +21\\ \hline 42\end{array}$. The trick here is that instead of thinking: 1 and 1 are 2, 20 and 20 are 40, 2 and 40 are 42; he can think: 1 and 1 are 2, 2 and 2 are 4.

From this trick of adding numbers in reverse he can develop a similar trick in multiplication. For $\begin{array}{r}21\\ \times\underline{2}\end{array}$ he can now discover that he can think: two 1's are 2, two 2's are 4. The written computation then looks like this, $\begin{array}{r}21\\ \times 2\\ \hline 42\end{array}$.

4. *Multiply a 2-digit number by a 1-digit number; the product of the ones exceeding 9.*

This type of computation can be performed using the same techniques as the preceding step.

$$\begin{array}{cc} \text{(a)} & \text{(b)}\\ \begin{array}{r}37\\ \times 4\\ \hline 120\\ 28\\ \hline 148\end{array} \text{ or } & \begin{array}{r}37\\ \times 4\\ \hline 28\\ 120\\ \hline 148\end{array}\end{array}$$

The child thinks: Four 30's are 120 and four 7's are 28, or four 7's are 28 and four 30's are 120.

In (b) the child can be challenged to add the 20 (of the 28) and the 120 mentally, writing the computation like this, $\begin{array}{r}37\\ \times 4\\ \hline 148\end{array}$. The "carrying" trick can evolve from this procedure or it can be suggested by the "carrying" previously learned in addition $\begin{array}{r}{\scriptstyle 2}\\ 37\\ 37\\ 37\\ +37\\ \hline 148\end{array}$. Here we have the four 7's added to make 28, the 2 being added to the 12 of four 3's to make 14. Similarly, for $\begin{array}{r}37\\ \times 4\\ \hline 148\end{array}$, four 7's are 28, add the 2 mentally to the 12 of four 3's to make 14.

If a child finds it too difficult to "carry" mentally he should be permitted to *write* the two partial products before adding them:

$$\begin{array}{r}37\\ \times 4\\ \hline 120\\ 28\\ \hline 148\end{array} \quad \text{or} \quad \begin{array}{r}37\\ \times 4\\ \hline 28\\ 120\\ \hline 148.\end{array}$$

5. *Multiply a 1-digit number by a multiple of 10.*

The child can be introduced to this type of computation by reviewing and writing the 10's combinations (Step 1):

$$\begin{array}{r}1\\ \times 10\\ \hline \end{array} \quad \begin{array}{r}2\\ \times 10\\ \hline \end{array} \quad \cdots \quad \begin{array}{r}9\\ \times 10\\ \hline \end{array}$$

From this he can build the 20's, 30's, 40's, etc., until he thinks of the following tricks: (1) $\begin{array}{r}4\\ \times 20\\ \hline\end{array}$ has the same answer as $\begin{array}{r}20\\ \times 4\\ \hline\end{array}$ (Step 2) $\begin{array}{r}8\\ \times 70\\ \hline\end{array}$ has the same answer as $\begin{array}{r}70\\ \times 8.\\ \hline\end{array}$ (2) For $\begin{array}{r}4\\ \times 20\\ \hline\end{array}$ think: Two 4's are 8, affix the 0. For $\begin{array}{r}8\\ \times 70\\ \hline\end{array}$ think: Seven 8's are 56, affix the 0.

Facility in getting answers mentally to this type of computation can be developed by using either of these tricks.

6. *Multiply a multiple of 10 by a multiple of 10.*

From the following table of 10's

$$\begin{array}{r}10\\ \times 10\\ \hline 100\end{array} \quad \begin{array}{r}20\\ \times 10\\ \hline 200\end{array} \quad \cdots \quad \begin{array}{r}90\\ \times 10\\ \hline 900\end{array}$$

tables of 20's, 30's, 40's, etc., can be derived.

$$\begin{array}{r}10\\ \times 20\\ \hline 200\end{array} \quad \begin{array}{r}20\\ \times 20\\ \hline 400\end{array} \quad \cdots \quad \begin{array}{r}90\\ \times 20\\ \hline 1800.\end{array}$$

The child will sooner or later with or without the teacher's help hit upon the trick of multiplying and affixing zeros. For $\begin{array}{r}80\\ \times 90\\ \hline 7200\end{array}$ think: Nine 8's are 72 zero zero.

The principle of affixing zeros could be more firmly fixed in the children's minds if the following types of multiplication were reviewed:

$\begin{array}{r}7\\ \times 6\\ \hline 42\end{array}$ Six 7's are 42. (Step 1)

$\begin{array}{r}70\\ \times 6\\ \hline 420\end{array}$ Six 7's are 42, affix the 0 from 70. (Step 2)

$\begin{array}{r}7\\ \times 60\\ \hline 420\end{array}$ Six 7's are 42, affix the 0 from 60. (Step 5)

$\begin{array}{r}70\\ \times 60\\ \hline 4200\end{array}$ Six 7's are 42, affix the two 0's from 70 and 60. (Step 6)

7. *Multiply a multiple of 10 by a 2-digit number.*

Some children at this point will be able to multiply any 2-digit number by any 2-digit number by applying techniques already learned. For example, $\begin{array}{r}23\\ \times 46\\ \hline\end{array}$, can be broken down into $\begin{array}{r}20\\ \times 40\\ \hline\end{array}$ (Step 6), $\begin{array}{r}3\\ \times 40\\ \hline\end{array}$ (Step 5), $\begin{array}{r}20\\ \times 6\\ \hline\end{array}$ (Step 2), and $\begin{array}{r}3\\ \times 6\\ \hline\end{array}$ (Step 1) and the partial products added. However, some children may require two more intermediary steps: $\begin{array}{r}40\\ \times 36\\ \hline\end{array}$ and $\begin{array}{r}46\\ \times 30\\ \hline\end{array}$.

$$\begin{array}{r}40\\ \times 36\\ \hline 1200\\ 240\\ \hline 1440\end{array}$$
$30\times 40=1200$ (Step 6); $6\times 40=240$ (Step 2); $1200+240=1440$

$$\begin{array}{r}40\\ \times 36\\ \hline 240\\ 1200\\ \hline 1440.\end{array}$$
$6\times 40=240$; $30\times 40=1200$
$240+1200=1440$

8. *Multiply a 2-digit number by a multiple of 10.*

$$\begin{array}{r}46\\ \times 30\\ \hline 1200\\ 180\\ \hline 1380\end{array}$$
$30\times 40=1200$ (Step 6); $30\times 6=180$ (Step 5); $1200+180=1380$.

$$\begin{array}{r}46\\ \times 30\\ \hline 180\\ 1200\\ \hline 1380.\end{array}$$
$30\times 6=180$; $30\times 40=1200$;
$180+1200=1380$

Some children can be led to the discovery of the trick of multiplying 3 and 46 (Step 4) and affixing a 0: $\begin{array}{r}46\\ \times 30\\ \hline 1380\end{array}$

9. *Multiply a 2-digit number by a 2-digit number.*

Using techniques already acquired, these are some of the ways children can get answers to this type of computation:

(a) $$\begin{array}{r}84\\ \times 63\\ \hline 4800\\ 240\\ 240\\ 12\\ \hline 5292\end{array}$$
$60\times 80=4800$; $60\times 4=240$; $3\times 80=240$; $3\times 4=12$.

(b) $$\begin{array}{r}84\\ \times 63\\ \hline 12\\ 240\\ 240\\ 4800\\ \hline 5292\end{array}$$
$3\times 4=12$; $3\times 80=240$; $60\times 4=240$; $60\times 80=4800$.

(c) $$\begin{array}{r}84\\ \times 63\\ \hline 252\\ 5040\\ \hline 5292\end{array}$$
$3\times 4=12$; $3\times 8=24$; $24+1=25$ ("carrying" trick—Step 4)
Place 0 under the 2 (Step 8); $6\times 4=24$; $6\times 8=48$; $48+2=50$ ("carrying" trick—Step 8).

"Indenting," as can be seen from the above, is not necessary. Method (c) is a perfectly good method to use. If, for some reason, however, it is thought desirable for a child to learn to "indent," he can be taught to *imagine* that he is placing the zero in the second partial product. But whether he "indents" or not, he would have at this point some idea why it is reasonable to do so.

By proceeding gradually from one type of computation to another, by applying previously learned principles and techniques to each new step, and by looking for short cuts in computational processes, children can be helped to invent and discover time-saving tricks of computation. They are, through inductive teach-

ing, thus helped to view computational processes as being sensible and understandable. Arithmetic, then, would not be just an exercise in manipulating mysterious and meaningless symbolisms.

EDITOR'S NOTE. Professor Hickerson has presented a different sequence for learning "long multiplication." It is a sequence designed particularly for an inductive procedure which avoids "indenting" partial products until the very last step. It is supposed to yield greater understanding of the multiplication process. There is a good deal of "open territory" for argument in several areas of computation with particular reference to the placement of digits, especially zero. Mr. Hickerson's method may appeal to some pupils and may seem too cumbersome for others. His aim, to promote understanding, should be our common aim. What do readers think of his sequence and method? Should it be urged as one for all pupils or for pupils who have a Mr. Hickerson for a teacher?

Seventeenth Christmas Meeting

of the

National Council of Teachers of Mathematics

At Arkansas State College, State College, Arkansas

Sectional Meetings Dealing with Arithmetic

Thursday, December 27, 1956

Helping Children Estimate Answers	IDA MAY HEARD
Use of Graphs in the Arithmetic Program of Retarded Children	MARGARET WILLERDING
Is Preservice Training of Arithmetic Teachers Satisfactory?	RUTH GUTHRIE
Arithmetic Discussion Conference	ELEANOR WALTERS, AGNES GUNDERSON, AND JOSEPH STIPANOWICH
Arithmetic Laboratory	LYLE HANCE

Friday, December 28, 1956

Evaluating Methods of Teaching	LOUIS E. ULRICH
Testing Versus Drill on Number Facts	LAURA EADS
Basic Concepts of Mathematics from Arithmetic to Calculus	J. D. HAGGARD
What Can We Do to Improve the Teaching of Arithmetic?	MILDRED B. COLE

Saturday, December 29, 1956

Structure Arithmetic in Terms of the Things We Do with Number	FRANCIS J. MUELLER

In addition to the discussions listed above the meeting will feature a full program dealing with secondary school mathematics and one with college mathematics. Several general sessions will feature speakers such as Joseph Bidwell, Head of the Engineering Mechanics Department of General Motors and B. J. Newchurch, Assistant Director of Esso Research Laboratories. It will be an excellent conference with the Arkansas Council of Teachers of Mathematics as host. Let us partake of their hospitality.

What's in a Rhyme?

RUTH HODGES TUTTLE
Denver, Colorado

I DISCOVERED THE EFFECTIVENESS of using little poems and rhymes to help children acquire number meanings when Bobby was six years old. Bobby was the first of 1B's to have a birthday after school opened in September. It was a big event. A birthday! A thrill for every child in the room, for each had at least five birthdays behind him. And each child had either recently become six or was about to do so.

Yesterday Bobby was five. Today he is six. What did it mean to the children other than a party, gifts, candles on a cake? Nothing more, really. But perhaps it could. Perhaps six could become really meaningful on this birthday occasion.

I recalled seeing a poem "The Birthday Song" in the new *Instructor* I had just received. I read it to the children.

BIRTHDAY SONG*

"Today, today," the big clock ticks,
"Kathleen is six, Kathleen is six!
"It seems but no time, sakes alive,
Since she was five, since she was five.
"And only minutes, nothing more,
Since she was four, since she was four.
"Tick-tock! Tick-tock! How can it be
That Kathleen Ann is two times three?
"Such funny things she used to do
When she was two, when she was two!
"Tick-tock! How fast the hours have run,
Since she was one, since she was one!
"And now she's half of twelve! So quick
The birthdays fly! Tock-tick! Tock-tick!
"Kathleen is six, Kathleen is six,
Hurray! Hurray!" the big clock ticks.

—MARY LOUISE CHEATAM

We played with this delightful little poem for days. "Tick tock! How fast the hours have run, Since she was one, since she was one."

How does a one-year-old child act? How does he talk, walk, laugh, cry, eat? (Um-m! What wonderful pantomime and dramatic effects we got, some from actual experience with younger brothers or sisters, other purely from imagination.)

Other questions aroused by the poem were: What does the family need to do for a one-year-old that a six-year-old can do for himself? What things can a one-year-old do that he couldn't do at one month? Why do we tell a baby's age in months until his first birthday? Discussing these things led us to realize having birthdays meant something really special.

Two-, three-, four-, five- and six-year olds were discussed in much the same manner. Little dramatizations of these various ages pointed up the meaning and advantage of becoming older, especially becoming six, so that one could be in first grade.

One day Judy brought her very own "One to Six" photograph album to share with us. What fun that was—seeing how Judy looked as a baby even too small to have her picture taken alone. Then how she looked as she got more hair, as she learned to sit alone, to stand, to walk, to ride a tricycle, to push a doll buggy.

Judy's album gave us another idea. We could each make an album of self-portraits. We could show how we grew and what we learned to do as we got bigger and older. We could show how we thought we looked and what we thought we were. Our planning allowed several pages for each age level, one to six. At the end of each age section we would draw a birthday cake with candles to show how old we had become. Our first cake would be at the end of section one and would have one candle. Our last cake would have six candles.

Those albums were precious. Each child portrayed himself in the various ages in

* Reprinted by permission of *The Instructor*, Dansville, New York.

his own individual style, but all gave evidence of having a clear concept of the growth and development that growing older carries. Each showed that he understood two was more than one, three more than two, and so on.

Acquiring a concept is of course merely being able to make a generalization from previous experiences when those experiences are brought into focus through new experiences.

"The Birthday Song" was a delightful vehicle for bringing out the idea that age is measured by the number of years one has lived previously. Being six made Bobby feel a new importance. He was "half of twelve." We knew that for sure by manipulating 12 birthday candles and setting aside six for Bobby's age now, and counting on through the other six for the years ahead until he would be 12. There were 6 candles in each group when we had finished. Being six made sense to Bobby and to the other 1B's who just had reached or soon would reach that age.

Finding out about the number four was fun, too, because we had a little poem to focus our attention on that number. The poem was "Goldfish" by Aileen Fisher and it goes like this:

GOLDFISH*

I have four fish with poppy eyes
Awfully poppy for their size.
Perhaps they're poppy from surprise
For after frisking in the sea
Fish must think it queer to be
Looking through a glass at me.

There just happened to be four fish in our big fish bowl the day we read the poem. (A teacher can always arrange a little matter like that.) What a lot of discoveries we made after the poem aroused a fresh interest in the fish. The fish were very accommodating about grouping themselves. We got, over a period of time, groupings of 1 and 3, 2 and 2, and of 4. We discovered that 4 fish had 8 eyes in all, 3 fish had 6 eyes in all, 2 fish had 4 eyes in all and so on. We learned that a fish has a pair of eyes and that pairs can be counted by 2's, and counting by 2's is more expedient.

No doubt you have never played at being a fish with poppy eyes. So, unlike my 1B's, you've missed a lot of fun. Being a fish with poppy eyes or being the one to count the poppy eyes was equally alluring to the 1B's. Do you think they forgot easily that 2, 4, 6, 8, 10 meant 5 pairs of eyes when those eyes belonged to other 1B's who were making them so "poppy" they could scarcely count for laughing? Oh, no!

From the fun with the goldfish poem we learned the meaning of pair, the relationship of 2 to 4, to 6, to 8, to 10, and the meaning of even numbers. We learned that 10 can be counted by 1's or by 2's. When we're counting *pairs*, it's much better to count by 2's because a pair means 2 alike, and counting by 2's goes faster.

Other poems like Marion St. John's "How to Tell the Time" help us to discover that the clock is a tool for measuring time. Of course, we don't have the dandelion puffs like the fairies have, but we play the fairies' game with our play clocks. All the children must have a turn at being a fairy. They set the clock for any time they wish and say:

(*three*) o'clock
(*four*) o'clock
(four)'s the time for play.

With a repertoire of such poems, the primary teacher need never be at a loss for something to spark the arithmetic period, either to introduce or to give meaning to routine number work. Children and teacher alike find the practice both practical and pleasurable.

* By permission of the McBride Company, Inc., New York.

Teaching Verbal Problems in Arithmetic

HARRY PEELER
Pelham Junior High School, Pelham, N. Y.

WHAT IS A PROBLEM? A problem consists of a situation which has no ready-made solutions. An arithmetic problem is a special quantitative situation involving perhaps size or amount. It involves a special vocabulary—the language of numbers. And the solution is reached through use of special techniques—one or more of the fundamental processes. Morton refers to a problem as "an arithmetical situation in which the operation to be performed is not indicated but must be decided upon by the pupil."[1]

When do we begin to teach problem solving? As soon as the child recognizes the numerals? As soon as the child learns to read and write the numerals? As soon as he can read verbal statements? As soon as he knows the fundamental processes? The correct answer is not found in any of these. These answers imply prerequisites in his instruction. They imply that there is a break in the systematic learning process, and that at this point or that point problem solving is introduced. Such is not the case. As a matter of fact, the child has probably been solving problems in arithmetic before he enrolls in school. If he has been faced with some such difficulty as dividing 12 cookies equally among three friends and actually counted the cookies into three groups, then he solved a problem. In his discovery by counting he used no written numerals, symbols or words. Nor was the problem stated in words or written. In school, under the guidance of the teacher, he manipulates concrete objects and discovers and learns how to combine groups into a larger group and how to take one group of objects from another group. He actually is solving a problem, perhaps stated to him orally, as for example: "If we take three apples away from this group of eleven apples, how many apples are left?" From such discussion and practice he learns the counting of numbers and the relationships between numbers. In like manner, he discovers and develops the concepts of addition and subtraction. He then practices and systematizes his concepts and relationships without benefit of concrete objects. But his concepts about numbers are the result of thinking in the concrete problem situations. "Many, many relationships need to be learned as children grow toward maturity in mathematical thinking—relationships among numbers, relationships among processes, relationships among a great variety of aspects in problem situations. The use of concrete materials helps children see relationships, discover others, and grow toward more mature levels of thinking."[2] So, a meaningfull approach to problem solving calls for problem solving directed toward an outgrowth of generalizations, understandings, and number facts—using these tools to lead into other phases of problem solving. This transition from one phase of problem solving to another phase of higher order does not come automatically but must be planned for and directed by the teacher.

[1] Morton, R. L. "Teaching Arithmetic in the Elementary School," (N. Y.: Silver Burdett Co., 1938), Vol. II, p. 454.

[2] Clark, John R. and Eads, Laura K. "Guiding Arithmetic Learning," Yonkers, N. Y.: World Book Co., 1954), p. 16.

A Continuing Process

There literally is no point on the scale in the learning process when children begin to solve problems, nor is there any point in time when they begin. The progressive steps forward in problem solving are characterized not by gaps, jumps, or breaks but by continuity.

"Problem solving, then, is not a formal, special division of a mathematics program. It is a daily life experience of children, and, in the case of mathematical situations, it is a way of using mathematics to answer the questions that children really want to have answered."[3]

There are many research studies pointing out the difficulties children encounter in problem solving. Osburn's[4] analysis of 30,000 errors in problem solving indicates that 30% of the errors were due to failure to comprehend the problem, 20% were due to errors in fundamental processes, 10% were due to failure to respond to fundamental quantitative relations, and 20% were due to errors the cause of which could not be determined. Banting, Brueckner, Lazerte, Stone, and others have made similar studies, attempting to analyze the causes of failure in problem solving. The results of the many studies show lack of agreement as to the causes of difficulty.

There is a greater disagreement as to the remedial and diagnostic measures recommended to overcome the difficulties. Various "plans" or "methods" have been suggested, such as formal analysis, teaching the cues, teaching better the reading skills, graphic analysis, emphasis on fundamental skill processes, and others. However, Morton says: "It must be apparent to the reader that no one of the problem solving methods which has been tried under experimental conditions produces markedly superior results."[5] The only thing wrong with these formal methods is that they fail to get the job done. These methods fail to take into account that in problem solving we are dealing with the higher mental processes and that the thinking of a group of children cannot be channeled down one narrow passageway. A formal way of thinking which might aid one child might actually be a hindrance to another. Clark and Eads say: "For children, however, a pattern of steps or formulas for solving such problems will interfere with their learning to think out solutions when they are mature enough to do so."[6] It would seem that there is no "royal road" to teaching problem solving successfully.

Formal Methods Fail

In the first place, any attempt to reduce the teaching of problem solving to a formal method is to imply that the learning of verbal problems is a separate, integral part of the total process of the arithmetic organization. Most of the experiments in the analysis of difficulties which children encounter in solving verbal problems also imply this. Perhaps the research in problem solving has been based on a mechanistic concept of teaching problem solving and arithmetic. In a program of teaching arithmetic meaningfully, problem solving is not one separate part of the whole but is an element running through and intertwined with the whole. As concepts, skills, generalizations and understandings progress and grow so does ability in problem solving grow, and as problem solving grows so grow the skills, concepts and understandings. Wheat's[7] concept of the verbal problem is that it is a device to be used to clarify an idea which the pupil has begun to develop.

[3] "Mathematics for Boys and Girls," (Albany, N. Y.: University of the State of New York, 1955), p. 53.

[4] Osburn, W. F. "Corrective Arithmetic," (Boston: Houghton Mifflin Co., 1924) p. 38.

[5] Morton, R. L. "Teaching Arithmetic in the Elementary School," (N. Y.: Silver Burdett Co., 1938) vol. II, p. 476.

[6] Clark, John R. and Eads, Laura K. "Guiding Arithmetic Learning," (Yonkers, N. Y.: World Book Co., 1954) p. 259.

[7] Wheat, H. G. "The Psychology and Teaching of Arithmetic," (N. Y.: D. C. Heath & Co., 1937) p. 213.

In the second place, the attempt to apply a formal "method" to the teaching of arithmetic, verbal problems in particular, is mechanistic in itself. It implies that if a pupil falls behind in his ability to solve verbal problems that the teacher can bolster him up by having him solve more verbal problems using a formal process.

In a program of teaching problem solving meaningfully, the teacher conceives of her job as not how to teach children to solve problems but to teach children how to *think*. And to think, one must have something to think with—facts, concepts, meanings and understandings. The teacher does not set out to teach problem solving as such, or skills as such, but these are developed simultaneously along with meanings inherent in number relations and with ideas of procedure. One should not lag behind the other, but all are woven together to make the total process of learning and thinking.

There are two important things which determine how proficient a pupil might become in problem solving and the depth and breadth of his future achievement. One is his mental ability, which the teacher can do little about. The other factor is the richness and thoroughness of his background in understandings, concepts, skills and meanings. Here lies the real challenge to the teacher. The extent to which a teacher is cognizant of these things, plans carefully, and works for them, largely determines the reach and scope of her pupils in problem solving.

In order to help a child with his difficulty in solving problems, it is necessary to know just what his difficulty is. This certainly cannot be determined by formal tests or written work alone. It may be necessary to have the child repeat his thinking orally to the teacher. Diagnosis must cover the thought processes as well as written work. Diagnostic and remedial work must be on an individual basis for the most part. It has been fairly well demonstrated that problem solving based on understanding is superior to that based on formal, mechanistic methods, that the degree of understanding is a function of the kind of instruction given, and that that kind of instruction which best enables a child to organize his previous experiences is the kind we should strive for.

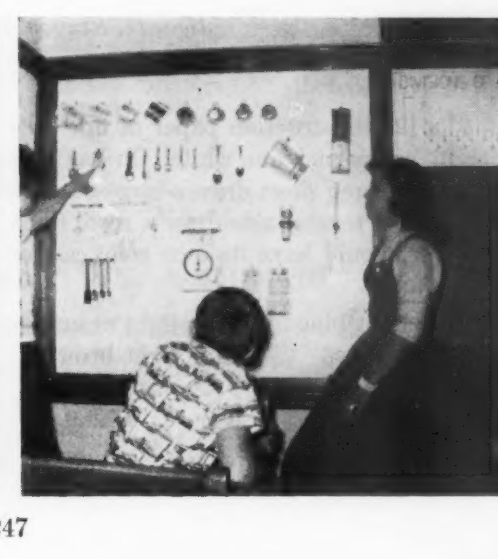
EDITOR'S NOTE. Mr. Peeler points out that written problems should not be considered a separate topic in arithmetic but rather they are a part of a continuous growth in learning. Our big job as teachers is to foster this growth toward higher levels and this is not easy because there is no simple formula for problem solving. We know all too little about how the human mind works and develops. Because of differing backgrounds of experience all children will not respond equally to the same stimulus and hence our better methods tend to treat the children as individuals each of whom must make his own discoveries and learning. But it is the teacher's role to stimulate and to foster and sometimes even to demand modes of thinking and working that are increasingly more mature. In solving problems we must remember that there probably is no one best method for all pupils and that the methods of thinking used in a visual-oral situation often should be different from the methods used in written work. We seem to be agreed that most pupils will learn to solve problems in arithmetic if we give them problems to do and help them develop insight with relationships and security with computations.

Measures Make Arithmetic Meaningful

GENE MCKEEN*
Grade 7, Whittier Jr. H. S., Lincoln, Neb.

"MEASUREMENT IN LIFE" was the title of one of our recent units in arithmetic. Prior to studying this unit, most of our class members had only vague ideas about the real value of measurement in everyday living. At the beginning of the unit, we discussed some of our own ideas as to where and how measurement was used throughout the world. We knew, of course, that carpenters measured with rulers and tapes and some in the class mentioned that the housewife used measurements when she used recipes in cooking and baking. As our study of measurements progressed, however, our knowledge of measurement and its value in many occupations took on more and more meaning. Our teacher, Mr. Norton, asked each member of the class to search the house for any measuring device that could be brought to school. Believe me, we soon discovered that the ruler and yardstick weren't the only measuring devices being used. Some of the class members brought instruments that most of us had never heard of or seen before. For example, three boys brought micrometers. Although most of the class had not heard of the micrometer before, all of us soon learned that the "mike" was a precision instrument used by many different people such as the machinist, geologist, metal worker, and factory worker. All of us learned the parts of the "mike" and also how to measure with it. We were really surprised to find that we could measure the thickness of a hair on our head; mine was 3/1000 of an inch thick. Some of the class members had difficulty in learning to read the instrument at first, but it wasn't long before all of us knew all about the "mike." In the process of finding out the interesting things about the "mike," we studied and learned how to work with decimal fractions. My sister had told me that decimals were hard, but I didn't think so even though arithmetic was one of my hardest subjects last year. In fact,

* Gene McKeen is a pupil who was in Mr. Monte S. Norton's class in arithmetic last year.



I thought decimals were fun and you certainly have to know them to use a micrometer.

The "mike" was only one of the measuring instruments we studied in our unit. In all, we counted over 175 measuring instruments that were displayed on our bulletin boards. We had a vernier caliper, inside and outside calipers, framing square, transit, opticians caliper, measuring cups and spoons, a feeler gauge, rulers, tapes, weighing devices, drafting devices, gauge blocks, and many many others. Several of us wrote to certain companies for charts and pamphlets on measurement. We even studied history. The history of measurements was one of the most enjoyable parts of the unit.

Most of us had been told how arithmetic was used, but the class found out first-hand that decimals, fractions, angles, measuring tables, and other phases of arithmetic are necessary for almost any job we might have to perform. In fact, at the end of the unit, we found it difficult to name a job in which arithmetic and measurement did not play an important role.

All of the class were pretty proud of our display of instruments that we had brought from home. In fact, we took several flash pictures of the bulletin boards. We felt proud too when several people from the university came to see our exhibit. The whole class voted to keep the exhibit up for our parents to see at open house at our school.

My friend, Steve, who sits next to me in class, said that fractions and decimals were becoming easier and easier for him to work with. I guess he summed up the feelings of the whole class when he said: "Gosh! who would have thought that learning arithmetic could actually be so much fun."

A Place-Value Game

LORENA W. HOLDER
John H. Reagan School, Dallas, Texas

Materials needed:

9″×12″ construction paper of nine different colors and one white. On one side of each colored sheet draw a large number and on the reverse side draw a zero. Each number should have its own color, as for example:

1 on light blue	7 on light orange
2 on yellow	8 on light brown
3 on pink	9 on dark orange
4 on red	0 on dark blue
5 on light green	, on white
6 on dark green	

Let's act out the writing and reading of 4,025. (The number 4,025 is written on the board.) The above number-sheets are passed out to the class. Pupils holding a 5 sheet raise a hand and one child is designated to go to the board and start the number. The pupil holds her number so the class can read the 5 and says,

(1) "My name is 5, I am in the units' place, my value is 5 units."

Then she turns her sheet over so the 0 on the reverse side is visible.

Then all pupils who have a sheet with 2 on it, raise a hand and a pupil is designated to place the 2. She holds her sheet so all can see and says,

(2) "My name is 2, I am in the tens' place, my value is 2 tens or 20 units." Then she turns her sheet over to show the reverse side.

Since all sheets have a zero on one side, only pupils having zero on both sides of their sheets hold up hand for recognition for the third position. When called on the pupil takes his place in line and says,

(3) "My name is zero, I am in the hundreds' place, I have no value, I am only a place holder."

Pupils with a comma on a white sheet now ask for recognition and when granted one takes the place in the line and says,

(4) "I am a comma, I am used to separate periods of three figures each."

A pupil holding a 4 sheet now takes a place in line and says,

(5) "My name is 4, I am in the thousands' place, my value is 4,000."

Up to this point, as each figure has given her *name*, *location*, and *place value*, all others are showing zeroes. When the last place figure has finished, she begins to read the entire number, others follow as their time comes, turning their sheets so the figures can be seen, and repeating their number and place value just as one person would read the entire number.

(5) continues, "When we all work together we are worth" (speakers by number in parentheses)

(5) "four thousand"
(4) comma does not speak
(3) "no hundred"
(2) "twenty"
(1) "five"

and we look like this: **4,025.**

Let's act out the writing and reading of *36,803,725* (Speakers by numbers in parentheses)

(1) My name is 5, I am in the units' place, my value is 5 units.
(2) My name is 2, I am in the tens' place, my value is 2 tens or 20 units.
(3) My name is 7, I am in the hundreds' place, my value is 700.
(4) I am a comma, I am used to separate periods of three figures each.
(5) My name is 3, I am in the thousands' place, my value is 3,000.
(6) My name is zero, I am in the ten-thousands' place, I have no value, I am only a place holder.
(7) My name is 8, I am in the hundred-thousands' place, my value is 800,000.
(8) I am a comma, I am used to separate periods of three figures each.
(9) My name is 6, I am in the millions' place, my value is 6,000,000.
(10) My name is 3, I am in the ten-millions' place, my value is 30,000,000.
(10) "When we all work together we are worth"
(10) "Thirty-"

(9) -"six million"
(8)
(7) "8 hundred"
(6)
(5) "Three thousand"
(4)
(3) "7 hundred"
(2) "twenty"
(1) "five."

36,803,725

Twenty-five Questions on Arithmetic*

Ben A. Sueltz
Cortland, N. Y.

1. What Should Be the Arithmetic of Grades I and II? Experiences with things involving size, shape, and number? Learning to think and work with number symbols as counting, comparing, adding, subtracting, etc.? Should we have definite goals for end of Grade I? For Grade II? Should we have "arithmetic periods"?
2. Is There an Age-Span when most children master mechanical facts and rote learning more readily than at other ages? Perhaps age 7–9? What implications are suggested?
3. Should We Decry rote learning? E.g. sequences such as alphabet and numbers; basic number combinations such as $5+9$, $16-7$, 3×8, $36\div 4$; facts of standard measures; etc.
4. With 30+ Pupils in a room and with the wide range of previous experience and "readiness" for arithmetic in Grades I, II, III, what can a teacher do to bring any semblance of organized arithmetic into a class?
5. Do We Waste a good deal of time by starting serious arithmetic when pupils are too young for optimum learning in both comprehension and rate of learning? Do we really know when it is best to learn reading, writing, arithmetic, cooperation, hygiene, etc.? What happened to the "Committee of Seven"? To Mr. Benezet, *et al?*
6. Do We Believe that the way in which something is encountered and learned may be even more important than the usual goals of learning? Do methods of learning and thinking transfer better than facts and skills? Are they retained longer?
7. What Is Grade Five? Is it merely the year after Grade Four? Should we have definite grade-goals? Should we have, (a) minimum goals? (b) desirable optimum goals? (c) limited maximum goals for each grade?
8. Do We Believe in individual pupil discovery leading to facts, principles, and generalizations, or do we prefer authoritarian learning in which the conclusion is stated by a book or teacher and the pupil "learns" it and relies principally upon memory for later use? How best can these be combined?
9. If We Believe in individualizing learning and the development of understanding with an element of discovery, then, are we also committed to permitting and encouraging pupils to work by different processes and procedures in the same room and on the same skill or process?
10. How Much of Functional Arithmetic outside of school is mental and oral and not encountered by printed words? What does this imply for us for school work? Are the better modes of mental and oral work usually different from those used for written work? Do we frown upon indirect but understood procedures?
11. What Is the Valid Role of "gadgets" and devices and the laboratory method in learning arithmetic? What dangers and misguidings does a teacher face?
12. Is Method in Subtraction a closed issue? Should all pupils in the same

* This list of questions has been compiled by the editor from numerous meetings with teachers, supervisors, and principals and from correspondence coming to The Arithmetic Teacher. It would be very helpful if different people about the country prepared short articles about each of the questions so these might be printed. Such articles should be more than casual personal opinion. They should be based upon experimental evidence.

room or in the same school use a common method? Should there be sequence or progression from one method to another in work with whole numbers? Should the method used with whole numbers be the same as used with fractions, denominate numbers, and signed numbers?

13. WHAT ABOUT CHECKING? Should all computations be checked? How? How can we check reasoning? What is the status of "casting out nines"?
14. DO WE BELIEVE that neatness, orderliness, and systematic procedure and setting down of work are desirable and worth cultivating? For all pupils? Can these values be so taught in arithmetic so they will transfer to other areas and also persist into later life?
15. IS ARITHMETIC MORE than a "tool subject" to have available when numbers are encountered? How can the cultural, emotional, social, and ethical aspects of living be served by or through arithmetic? How can arithmetic be used as a mental therapeutic?
16. IN SEVERAL SECTIONS of the country, it is reported that children just hate arithmetic. What are probable reasons for this? How can they be avoided? Corrected? Why do many adults suffer mental blockage when they face a simple mathematical task or proposition?
17. WOULD YOU BE WILLING to drop all but the very simple computation with common fractions from the intermediate grades, follow this with decimal fractions, and then in upper grades continue technical work with common fractions for those who can learn this readily? This was common practice over 100 years ago.
18. DIVISION IS DIFFICULT. Should division with one-figure divisors be thought multiplicatively? Should we be content to limit division to two-figure divisors for all pupils of less than average ability?
19. HOW CAN WE BEST TEST and evaluate such things as understanding of concepts and principles and the ability to sense and to use these in functional situations?
20. WHAT ABOUT DRILL? Is it important? How can the needed amount be lessened from prevailing practice? How much "over-learning" is desirable? How about drill on ideas, thinking, and understanding, judgment, etc.?
21. WHAT ARE WE going to do with the really able pupils in any grade? Let them proceed to the work of the next grade? Give them more of the same work, only harder? Give them depth of insight? What will you do with the very slow learners?
22. IN MANY SCHOOLS results in arithmetic are very sad at the end of Grade Six, at the end of Grade Eight, and at high school graduation. What should be done about this? Have we evidence that any reasonably typical school achieves just about as much as it honestly tries to achieve over a period of years?
23. WHAT ARE GOOD METHODS of learning and relearning in Grades 7 and 8 of those things which were not well learned in previous grades? Are there better methods for learning to add at Grade 7 level than those used in Grade 3?
24. IS NOT PERCENTAGE one of the most worthwhile topics in arithmetic? Does this include "case III"? How can percentage be learned in school so that its values and uses will persist and can be extended?
25. WHAT ABOUT A GOOD COURSE in arithmetic in the high school in perhaps Grade 11 or 12? Should this be "consumers arithmetic"? Business arithmetic? or just good basic arithmetic with applications? What obligations have we to the American housewife who is the householder, cook, purchasing agent, etc. of the family?

The Work of the National Council

THE NATIONAL COUNCIL of Teachers of Mathematics is deeply concerned with and interested in the teaching of arithmetic. It recognized that arithmetic is the basis of all succeeding study of mathematics. The Council has made sure that problems on teaching arithmetic will be given adequate attention by including on its Board of Directors a vice president at the elementary school level. Usually several other members of the Board of Directors are also directly involved in the improvement of instruction in arithmetic. There now exists a definite program of service to the instruction in elementary school mathematics. This program consists of three major areas, namely, publications, meetings, and committees.

Publications

The Council publishes this journal, THE ARITHMETIC TEACHER, which is concerned only with the problems of instruction and teacher training in mathematics at the elementary school level. The value of the articles published in past issues and to be published speak for themselves. The Council has published three yearbooks of special significance to arithmetic teachers. They are the Tenth Yearbook, *The Teaching of Arithmetic;* the Sixteenth Yearbook, *Arithmetic in General Education;* and the Twenty-first Yearbook, *The Learning of Mathematics*. Each of these books contains a wealth of practical considerations for every classroom teacher. In addition the Council has published a number of small publications called the *How-To* series. Three of these publications are of direct value to arithmetic teachers. They are, *How to Use a Bulletin Board, How to Develop a Teaching Guide*, and *How to Use Films and Filmstrips*. More of these booklets will be published in the near future. Finally, a new yearbook on the teaching of arithmetic, designated for publication in 1959, is now well underway. The chairman of the committee producing this Yearbook is Dr. Foster E. Grossnickle and the other members are all outstanding experts on the teaching of arithmetic. This book promises to be one of the best yet.

Meetings

The Council sponsors several meetings every year, but the high light is the National Convention held in the Spring. The next convention is at Philadelphia, March 28–30, 1957. There will be sections on arithmetic Thursday afternoon, Friday morning and afternoon, and Saturday morning and afternoon. The list of speakers includes nationally recognized leaders such as Charlotte Junge, Nathan Lazar, Agnes Gunderson, Maurice Hartung, Fred Weaver, John Clark, Ben Sueltz, etc. The topics include: Language Difficulties; Gifted Children; Factors Retarding Improvement; Mathematical Training of Teachers; Pedagogical Training of Teachers; Developing Flexibility of Thinking and Performance; Materials and Devices for the Teacher, etc. No teacher who would be up to date can miss this convention where the latest available knowledge on arithmetic instruction will be presented.

Committees

The National Council has more than twenty committees at work on important aspects of mathematical education. Of special interest to arithmetic teachers is the Committee on the Elementary School Curriculum in Mathematics with Professor J. Fred Weaver as chairman. This committee is reappraising the entire curriculum in arithmetic toward developing a more effectively differentiated curriculum, and a more effectively articulated curriculum. The place of concepts, skills, memory work, meanings, sequential structure, and levels of learning in the arithmetic program are under investigation. Research and the report of this committee will eventually bring to the National Council definite recommendations for a better elementary school program in mathematics.

There is also a committee on Teacher Recruitment, Certification, and Standards headed by Dr. Kenneth Brown of the U. S. Office of Education. This committee will attempt to set a level of competence on the part of teachers that is reasonably practical and yet of professional worthiness. Other committees of interest to the elementary school teacher are the Committee on International Relations which will bring the Council news of instruction in other countries, the Supplementary Publications Committee which will be glad to have your suggestions for pamphlets that arithmetic teachers feel will aid them in their daily teaching, a Television Committee which is investigating the ways and values of this media of instruction, and a Nomination Committee for the 1958 elections, which is eager to have your suggestions for nominees for offices in the Council.

The Council is your servant. We want you to make full use of it.

HOWARD F. FEHR, *President*

THE ARITHMETIC TEACHER

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